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SCALE RATIOS IN THE STANDARD MODEL*

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ABSTRACT: We review the present knowledge of the Standard Model that is relevant in formulating its possible short distance extensions. We present different scenarios in terms of the Higgs mass, the only unknown parameter of the model. We concentrate on the many small numbers in the model and suggest generic methods to reproduce these numbers in terms of scale ratios, applying see-saw like ideas to the breaking of chiral symmetries.

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The Standard Model is described in terms of a mere twenty parameters, counting Newton's constant. The challenge to theorists is to devise *the* extension of the Standard Model which explain not only the number of parameters but their values as well. Any extension will predict many new phenomena at shorter distances. There are many candidates for extending the Standard Model, but none have so far distinguished themselves by reproducing the *values* of the parameters, not even their multiplicity. Thus it is timely to review the types of extensions which might generically explain the observed patterns, before plunging in detailed models.

The Standard Model is described by three dimensionless gauge couplings α_1 for the hypercharge $U(1)$, α_2 for the weak isospin $SU(2)$, and α_3 for QCD. QCD itself predicts strong CP violation, parametrized by a fourth dimensionless parameter $\overline{\theta}$.

The Higgs sector yields two parameters, a dimensionless Higgs self-coupling, and the Higgs mass. The self coupling is expressed in terms of the scale of electroweak breaking, which is directly "measured" as the Fermi coupling. The value of the Higgs mass is the only parameter that has not yet been determined from experiment.

The Yukawa sector of the model yields the nine masses of the elementary fermions, which are in turn expressed as nine dimensionless Yukawa couplings multiplied by the electroweak order parameter. This sector also contains three mixing angles which account for interfamily decays, and one phase which describes CP violation in these decays.

Let us start by discussing the dimensionful parameters. The most important is Newton's constant which sets the scale. All fundamental questions concerning dimensionful parameters should be posed in terms of the Planck scale (10^{-33} cm, or 10^{19} GeV). Together with the other two fundamental constants, it sets a truly natural system of units. The second most striking one is the value of the electroweak order parameter, the inverse square root of the Fermi constant, in terms of the Planck mass

$$\frac{G_F^{-1/2}}{M_{Pl}} \sim 10^{-17} \ .$$

There is no satisfactory explanation for this small parameter. All proposed extensions have strived to explain the value of this number. One class of theories, generically called technicolor, has proposed the existence of strong new forces just beyond electroweak scales;

this yields a natural explanation of this parameter, but fails to explain the values of the fermion masses. Another class of theories postulates the existence of another type of symmetry, supersymmetry[1]. There, the electroweak order parameter is related to another small parameter, the order parameter of supersymmetry breaking. This may not seem very economical, but it is remarkable that supersymmetry breaking automatically generates electroweak breaking[2] in a wide class of theories. Thus it appears that there is nothing gained nor lost. The ideas of technicolor can then be successfully applied to supersymmetry breaking, by means of gaugino condensation, without the problem of fermion masses. Thus many believe that supersymmetry provides the best hope for explaining both the electroweak breaking scale and the value of the fermion masses.

All quark and charged lepton masses break weak isospin by half a unit, along $\Delta I_W = \frac{1}{2}$, with the same quantum numbers as the electroweak order parameter, which gives the W-boson its mass. It is thus natural to form the dimensionless ratio

$$\frac{m_t}{M_W} \sim \mathcal{O}(1) \ ,$$

which has a natural value. However there are other quark masses for which these ratios are much smaller,

$$\frac{m_{u,d}}{M_W} \sim 10^{-4} \ ; \qquad \frac{m_s}{M_W} \sim 10^{-3} \ ; \qquad \frac{m_c}{M_W} \sim 10^{-2} \ ; \qquad \frac{m_b}{M_W} \sim .05 \ .$$

Similarly for the charged leptons

$$\frac{m_e}{M_W} \sim \mathcal{O}(10^{-5}) \ ; \qquad \frac{m_\mu}{M_W} \sim \mathcal{O}(10^{-3}) \ ; \qquad \frac{m_\tau}{M_W} \sim .02 \ ,$$

range from the tiny to the small.

The neutrino masses are predicted to be exactly zero in the standard model only because of the global lepton number symmetries. However neutrino masses, if they were to be non-zero, would break weak isospin by one unit, that is have $\Delta I_W = 1$ values. Experimental limits on neutrino masses indicate that they are at most extremely small. For instance,

$$\frac{m_{\nu_e}}{M_W} < 10^{-17} \ .$$

Interestingly, this is reflected by the fact that weak isospin shows no sign of having been broken in that direction. We should mention that the masslessness of the photon and the gluons is deemed natural since protected by a gauged symmetry.

The values of the three gauge parameters are known to great accuracy. Because of endemic problems associated with strong QCD, that coupling is the least well known.

Given all these parameters, we can extrapolate the Standard Model to shorter distances, using the renormalization group. The most interesting effect occurs in the extrapolation of the three gauge couplings. We normalize the hypercharge coupling as if it were part of a non-Abelian group in which the standard model groups fit snuggly ($SU(5),\ SO(10), E_6$[3]). We find that the hypercharge and weak isospin couplings meet at a scale of 10^{13} GeV, with a value $\alpha^{-1} \approx 43$. We also find that at that scale, the QCD coupling is much larger, $\alpha_3^{-1} \approx 38$. Thus, although the quantum numbers indicate a possible unification into a larger non-Abelian group, the gauge coupling do not follow suit in this naive extrapolation. Historically of course, before the couplings were known to this accuracy, it was believed that all three did indeed unify in the ultraviolet. In any case, the lack of observed proton decay restricts the scale of unification to above 10^{16} GeV. One can still say that in the ultraviolet, the values of these couplings is less disparate than at experimental scales. Similarly, nothing spectacular occurs to the Yukawa couplings. For instance, the botton quark and τ lepton Yukawa couplings meet around 10^9 GeV, but diverge in the deeper ultraviolet.

The situation is potentially more interesting in the Higgs sector because of the renormalization group behavior of the Higgs self coupling[4]. We can consider two cases, depending on the value of the Higgs mass.

If the Higgs mass is below 150 GeV, the self-coupling turns negative at shorter diarances. This results in an unbounded potential, and instability of the standard model beyond the scale at which it changes sign. For example, using the recently measured value of the top quark mass, we find that a Higgs mass of 120 GeV would mean instability setting in at 1 TeV. In such case, new particles with masses commensurate with that scale must exist to stabilize the theory. This is exactly what happens in the supersymmetric extension of the Standard Model. One may envisage other stabilizing schemes without

supersymmetry, but it is just the most tractable.

If the Higgs mass is above 200 GeV, the self-coupling rises dramatically towards its Landau pole at a relatively low energy scale. This only means that we lose perturbative control over the theory. It sets an upper bound on the Higgs mass since there is no evidence of strong coupling at our scale. This is called the triviality limit because, looked at from the other side, it drives the self-coupling to zero in the infrared. However we know that the coupling is *not* zero for the standard model; thus strong coupling must happen. In all likelihood, this means that the Higgs is a composite; an example of this view is the technicolor scenario where the Higgs is a condensate of techniquarks.

Within a tiny range of intermediate values for the Higgs mass, the instability and triviality bounds are pushed to scales beyond the Planck length. In this case, there is no Standard Model prediction of new physics, except for the usual caveats associated with quantum gravity. Then we should view the Planck mass as the physical cut-off of any theory at lower energies. It is instructive to see what happens to the various Standard Model parameters in terms of the Planck cut-off.

The most striking behavior is that the renormalization of the Higgs mass is proportional to the cut-off itself. This does not make it natural to envisage a light Higgs with such an enormous renormalization. Thus even if the Higgs mass does not demand new physics below Planck mass, it makes for a pretty *ad hoc* theory. We can contrast the situation with fermion masses. Their dependence on the cut-off is only logarithmic. The reason is that a fermion mass is natural in the sense that by setting it to zero, one gains a chiral symmetry that is respected by quantum corrections. This allows for a protection mechanism which results in a weak cut-off dependence.

Supersymmetry avoids the naturalness problem in the following way: it links any fermion to a boson of the same mass, so that in the limit of exact supersymmetry, the boson mass is also protected by the chiral symmetry hitherto associated with the fermion. This is enough protection to assure, even after supersymmetry breaking, a mass for the Higgs that is commensurate with the scale of supersymmetry breaking.

This might seem to be small progress, since a new symmetry has been introduced to relax the strong cut-off dependence. That new symmetry has to be broken itself at a small

scale. Indeed, in order to reproduce the value of Fermi's constant, we must be able to obtain

$$\frac{V_{SUSY}}{M_{Pl}} \sim 10^{-15} ,$$

where V_{SUSY} is the supersymmetry breaking order parameter. Assume for a moment we know how to do this, and see if we have gained anything.

The first thing is that the gauge couplings seem to be much closer to unification, and at a scale not invalidated by proton decay bounds. One finds that the hypercharge and weak isospin couplings meet at a scale of the order of 10^{16} GeV, with a value $\alpha^{-1} \approx 25$. In this case, however, the QCD coupling is much closer to, if not right on the same value(5). It may still be a shade higher than the others, with $(\alpha^{-1} - \alpha_3^{-1}) \leq 1.5$.

The second thing is that with this value, and suitable boundary conditions at or near Planck mass, the renormalization group drives one of the Higgs masses to imaginary values in the infrared. This in turns triggers electroweak breaking, made possible only because of the large top quark mass.

It is significant that the extension to supersymmetry yields a model with no couplings that blow up below Planck mass. For example, the Higgs self-coupling is replaced by gauge couplings which are ultraviolet-tame. However, the Higgs mass is not arbitrarily high in the minimal extension. At tree-level, it is predicted to be below the Z-mass, but it suffers large radiative corrections due to the top Yukawa coupling, raising it above the Z, but not by an arbitrarily large amount(6).

This general scheme allows us to study the pattern of fermion masses at these shorter distances. It is interesting that there are more regularities with supersymmetry than without. For instance, the bottom quark and τ masses seem to unify at or around 10^{16-17} GeV(7).

As we have seen, most of the parameters yet to be explained are to be found in the Yukawa sector. With supersymmetry, the observed pattern of Yukawa couplings can be extrapolated all the way to or near Planck length. The hope is that at that scale, where things are supposed to be simpler, there might emerge some patterns not recognized at lower energies.

The most striking aspect of the fermion masses is that only the third family has sizeable masses. Thus it is natural to consider theories where the Yukawa matrices are simply of the form

$$\mathbf{Y}_{u,d,e} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_{t,b,\tau} \end{pmatrix} .$$

These matrices imply an enormous global chiral symmetry in each sector of the $U(2)_L \times U(2)_R$. There is of course the hierarchy between the bottom and top quark masses which must also be explained. In the $N=1$ model, it is related to another parameter which comes from the Higgs sector, the ratio of the *vev* of the two Higgs. We do not concern ourselves with it here. Thus the question of interest is really why are the other two families so light? In order to gain some perspective on this question, let us examine one well-known case in which small numbers are naturally generated, the see-saw mechanism[8].

The Standard Model neutrino Majorana mass matrix is zero. How do we fill the zeros, which are protected by lepton number conservation? They can be filled only if lepton symmetry is broken.

What happens in the see-saw mechanism is that the usual neutrinos are mixed with new electroweak singlet neutrinos. This gives them the same lepton numbers. Then the lepton numbers are broken by giving these neutrinos a mass M, which breaks lepton number at the same scale M. Upon diagonalization, this generates an entry in the mass matrix which is depressed from its expected value by the ratio of scale $\frac{m}{M}$, where n is the typical electroweak scale.

Let us analyze the charged Yukawa matrices in the same way. The zeros of the Yukawa matrices are protected by chiral symmetries. Thus we first couple the massless fermions with fermions with similar quantum numbers. This shares the chiral symmetries with the new fermions. Then we assume these new fermions, being charged have a vector-like partner (this differs from the neutral sector), and that they can acquire $\Delta I_W = 0$ mass at a new scale M. This mass breaks the chiral symmetry. Upon diagonalization, this fills some of the entries.

Consider a generic model with 3 left-handed fields f_i, f_3. Assume that the only tree level Yukawa involving the chiral fields is of the form $f_3 f_3 h$, where h is a Higgs which can

break electroweak symmetry. In the absence of any other couplings, this leaves us with a left-handed $U(2)$ symmetry acting on $f_{1,2}$. Thus if this symmetry is not broken, these fields will forever remain massless, at least in perturbation theory. We have to find a way to break this chiral symmetry.

To do this, let us add to the model N vector-like families $F_a \oplus \overline{F}_b$. After marrying off the left and right handed fields, we still have three chiral families. Such a situation generically arises in superstring compactifications where vector-like particles are readily available. The importance of these fields is that they can be used to break the chiral symmetry. First we observe that they can have $\Delta I_W = 0$ masses which do not break electroweak symmetry, of the form $M_{ab}\overline{F}_a F_b$. These terms break the chiral symmetries associated with the vector-like families. In order to relate the two types of chiral symmetries, we must couple these to the f_i. There are two types of such terms. The first is itself vector-like, and can occur at the large scale: $f_{i,3}\overline{F}_a$. The second type is of the form $f_{i,3}F_a h$, and breaks the electroweak symmetry, of the same type found in the see-saw mechanism. We can of course consider chiral operators of the form $F_a F_b$, and their conjugates, as well. However these might yield extra light particles in the spectrum, since these operators have electroweak quantum numbers. Thus we do not include them.

For instance, with one vector-like family, we may consider a Lagrangian of the form

$$f_3 f_3 h + (f_1 + f_2 + f_3 + f_4)\overline{F} H \ ,$$

where h is the electroweak breaking Higgs, and H is in the $\Delta I_W = 0$ sector. The fermion fields are all left-handed and refer to families. We have seven fields, and five terms in the Lagrangean, leaving us with two symmetries, which are both broken when h and H get vacuum values. Upon diagonalization of the mass matrix, we find two tree-level zero eigenvalues, but they are unprotected by chiral symmetries, and will be radiatively corrected. This model can be easily implemented in $SO(10)$ and E_6. However the symmetries do not forbid couplings such as $f_1 f_3 h$; one has to appeal to supersymmetry to explain the naturalness of these zeros.

It is a matter of model building to come up with specific arrays of vector-like particles which reproduce the family hierarchy. It is not easy to come up with such models, in the absence of extra symmetries.

From the point of view of a low energy effective theory, where the effect of the massive vector-like particles have been integrated out, the zeros will be filled ny non-renormalizable effective operators of the form

$$f_i f_j h \left(\frac{K}{M} \right)^{n_{ij}} ,$$

where h is the usual Higgs field, and K is a combination of Higgs doublets which can get non-zero vacuum value, and M is a large mass. The exponents n_{ij} may be determined by symmetry. In order to produce a small coefficient, the ith and jth fermions need to go through a number of intermediate steps to interact. The larger the number steps, the larger n_{ij}, and the smaller the entry in the effective Yukawa matrix. This approach was advocated long ago by Froggatt and Nielsen(9).

One may take the point of view that these non-renormalizable operators come from physics beyond the Planck scale, in which case, the question is relegated to one of classifying the possible non-renormalizable operators, without having to say how they are generated.

Clearly much work needs to be done before any successfull model of this type is devised, but we believe that this is a correct framework to analyze the Yukawa patterns.

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